Possibilistic Conditional Preference Networks

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Abstract. The paper discusses the use of product-based possibilistic networks for representing conditional preference statements on discrete variables. The approach uses non-instantiated possibility weights to define conditional preference tables. Moreover, additional information about the relative strengths of symbolic weights can be taken into account. It yields a partial preference order among possible choices corresponding to a symmetric form of Pareto ordering. In the case of Boolean variables, this partial ordering coincides with the inclusion between the sets of preference statements that are violated. Furthermore, this graphical model has two logical counterparts in terms of possibilistic logic and penalty logic. The flexibility and the representational power of the approach are stressed. Besides, algorithms for handling optimization and dominance queries are provided.

1 Introduction

Since the direct assessment of a preference relation between elements of Cartesian products is usually not feasible, current work in preference modeling aims at proposing compact preference models achieving a good compromise between elicitation easiness and computational efficiency. Conditional preference networks (CP-nets) [4] are a popular example of such setting. However, in spite of their appealing graphical nature, CP-nets may induce debatable priorities between decision variables and lack a logical counterpart. Symbolic possibilistic logic bases stand as another approach to represent preferences [9]. This setting overcomes the above mentioned CP-nets limitations. Moreover, it leaves complete freedom for stating relative priorities between variables. But, it is not a graphical model.

This paper explores the representation of preferences by possibilistic networks, outlined in [1] and establishes formal results about them. This approach preserves a possibilistic logic representation, while offering a graphical compact format convenient for elicitation.

The paper is organized as follows. Section 2 provides a formal definition of product-based possibilistic network with symbolic weights, and shows the nature of its preference ordering. Section 3 deals with the case of Boolean decision variables and provides two logical counterparts of this model, in possibilistic logic and in penalty logic. Section 4 discusses optimization and dominance queries.

2 Possibilistic preference networks

This section provides a short refresher on possibilistic networks, and then describes how conditional preferences can be encoded by a possibilistic network. Moreover, we show that the use of product-based conditioning leads us to define a preference ordering that amounts to compare vectors by a symmetric extension of the Pareto ordering.

2.1 Background on possibilistic networks

Possibility theory can be used for representing preferences. It relies on the idea of a possibility distribution π, which is a mapping from a universe of discourse Ω to the unit interval $[0, 1]$. Possibility degrees $\pi(\omega)$ estimate how satisfactory the solutions ω is. Since alternative choices are usually described by means of several decision variables, we need to manipulate possibility distributions on a Cartesian product $\Omega = D_{A_1} \times \ldots \times D_{A_N}$. Namely, each composite decision $\omega = (a_1, \ldots, a_N)$ (denoted for short by $a_1 \ldots a_N$), corresponds to an instantiation of the N variables $V = \{A_1, \ldots, A_N\}$, where A_i ranges on domain $D_{A_i} = \{a_{i1}, \ldots, a_{in}\}$. If $U \subseteq V$, then $\omega[U]$ denotes the restriction of solution ω to variables in U. Conditioning is defined from the Bayesian-like equation $\pi(A_i, A_j) = \pi(A_i|A_j) \otimes \pi(A_j)$ [3], where $\otimes$ stands for the product in a quantitative (numerical) setting or for min in a qualitative (ordinal) setting. Thus, the joint possibility distribution on Ω can be decomposed using conditional possibility distributions by means of the *chain rule* $\pi(A_1, ..., A_N) = \bigotimes_{i=1..N} \pi(A_i \mid Pa(A_i))$ where the set $Pa(A_i) \subseteq \{A_{i+1}, \ldots, A_N\}$ forms the parents of A_i. A_i is conditionally dependent on its parent variables only. This decomposition has a graphical counterpart, called possibilistic network, where each node encodes a variable related to each its parents by a directed arc. In the following, we use possibilistic networks for representing preferences (rather than uncertainty as it has been the case until now).

2.2 Preference specification

The user is supposed to express his preferences under the form of comparison statements between variable instantiations, conditional on some other instantiated variables. Therefore, in the particular case of Boolean variables, we deal with preferences of the form: "I prefer a to $\neg a$" if the preference is not conditioned, and of the form "in the context where c is true, I prefer a to $\neg a$" if conditioned. More formally,

Definition 1 A preference statement *s is a preference relation between values $a_{ik} \in D_{A_i}$ of a variable A_i, in the form of a complete preorder, i.e., we have only* 2 *different cases:*
i) $u_i : a_{ik} \succ a_{im}$: in the context u_i, a_{ik} is preferred to a_{im};
ii) $u_i : a_{ik} \sim a_{im}$: in the context u_i, the user is indifferent between a_{im} and a_{ik}, where u_i is an instantiation of all variables that affect the user preferences concerning the values of A_i. If $u_i = \emptyset$, then A_i is an independent variable.

The running Example 1, inspired from [4], illustrates such preference statements.

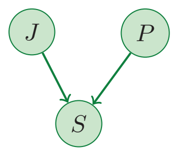
Example 1 *Consider a preference specification about an evening dress over 3 decision variables $V = \{J, P, S\}$ standing for jacket, pants and shirt respectively, with values in $D_J = \{Red\ (j_r),\ Black\ (j_b)\}$, $D_P = \{White\ (p_w),\ Black\ (p_b)\}$ and $D_S = \{Black\ (s_b),\ Red\ (s_r),\ White\ (s_w)\}$. The conditional preferences are given in Table 1. Preference statements (s_1) and (s_1) are unconditioned. Note that the user is indifferent between the values of variable S in context $u_j = j_b p_w$.*

(s_1) $j_b \succ j_r$
(s_2) $p_b \succ p_w$
(s_3) $j_b p_b$: $s_b \succ s_r \succ s_w$
(s_4) $j_b p_w$: $s_w \succ s_b \succ s_r$
(s_5) $j_r p_b$: $s_r \succ s_b \succ s_w$
(s_6) $j_r p_w$: $s_b \sim s_r \sim s_w$

Table 1. Conditional preference specification

$\pi(j_b)$	$\pi(j_r)$
1	α

$\pi(p_b)$	$\pi(p_w)$
1	β



$\pi(.\|.)$	$j_b p_b$	$j_b p_w$	$j_r p_b$	$j_r p_w$
s_b	1	δ_3	δ_5	1
s_r	δ_1	δ_4	1	1
s_w	δ_2	1	δ_6	1

Fig. 1. A possibilistic preference network

2.3 Graphical possibilistic encoding of preferences

As already said, conditional preference statements can be associated to a graphical structure. In this paper, this graphical structure is understood as a possibilistic network where each node is associated with a conditional possibility table used for representing the preferences. For each particular instantiation u_i of $Pa(A_i)$, the preference order between the values of A_i stated by the user will be encoded by a local conditional possibility distribution. So, each node A_i is associated with a conditional preference table. We call this model possibilistic conditional preference network (π-*Pref net* for short).

Definition 2 *A* possibilistic preference network (π-Pref net) *ΠG over a set $V = \{A_1, \ldots, A_N\}$ of variables is a preference network where we associate to each node $A_i \in V$ a possibilistic preference table (π_i-table for short), such that to each instantiation u_i of $Pa(A_i)$ is associated a* symbolic *conditional possibility distribution defining an ordering between the values of A_i:*

- *If $a_{ik} \prec a_{im}$ then $\pi(a_{ik}|u_i) = \alpha, \pi(a_{im}|u_i) = \beta$ where α and β are non-instantiated variables on (0, 1] we call* symbolic weights*, and $\alpha < \beta \leq 1$;*
- *If $a_{ik} \sim a_{im}$ then $\pi(a_{ik}|u_i) = \pi(a_{im}|u_i) = \alpha$ where α is a symbolic weight such that $\alpha \leq 1$;*
- *For each instantiation u_i of $Pa(A_i)$, $\exists\ a_i \in D_{A_i}$ such that $\pi(a_i|u_i) = 1$.*

Let $\mathcal{C}$ be the set storing the constraints existing between the symbolic weights introduced as above. This set can be completed by additional constraints, directly provided by the user.

By a symbolic weight, we mean a symbol representing a real number whose value is unspecified. However, inequalities or equalities between such unspecified values may be enforced, as in Definition 2, between conditional possibilities, or independently stated in $\mathcal{C}$. Since the symbolic weights stand for real numbers, relations $\leq$ and $<$ are transitive.

As usual in possibilistic networks, the normalization condition (expressed by the third item in Definition 2) is crucial for conditional possibility distributions. For example, consider a variable A such that $D_A = \{a_1, a_2, a_3\}$ and its context instantiation u, and assume that the user is indifferent between the values of A in that context. Then, $\pi(a_1|u) = \pi(a_2|u) = \pi(a_3|u) = \alpha$. Then, in order to satisfy normalization, α should be equal to 1 (see Example 1). In addition to the preferences encoded by a π-Pref net, additional constraints in $\mathcal{C}$ can be taken into account. Such constraints may, in particular, reflect the relative importance of variables by making all preferences associated to a variable more imperative than the ones associated to another variable, or express the relative importance of preferences associated to different instantiations of parent variables of the same variable. In the case one can not infer any relation between two weights by transitivity (distinct from 1), we consider them as incomparable.

Example 2 *Given the preference statements of Example 1, we can associate the possibilistic preference network ΠG in Figure 1 encoding the user preference over V. The preference statements corresponds to the set of constraints $\mathcal{C} = \{\delta_2 < \delta_1,\ \delta_4 < \delta_3,\ \delta_6 < \delta_5\}$. Consider, for instance, the preference statement s_6. Due to the normalization condition, $\pi(s_b|j_r p_w) = \pi(s_r|j_r p_w) = \pi(s_w|j_r p_w) = 1$.*

In this work, we explore the properties of possibilistic networks where conditioning is based on product. It has sometimes a greater discriminating power than the minimum operator, in the sense that $\alpha \cdot \beta < \alpha$, while we only have $\min(\alpha, \beta) \leq \alpha$. For instance, if $\alpha = \gamma < \delta < \beta$ then min considers (α, β) and (γ, δ) as equal, while we have $(\alpha, \beta) > (\gamma, \delta)$ with the product. However, if $\alpha < \gamma < \delta < \beta$ then $(\alpha, \beta) < (\gamma, \delta)$ with the min while the product operator fails to order them.

Example 3 *Let us consider the possibilistic preference network of Example 2. Using the chain rule, we obtain the following symbolic joint possibility distribution: $\pi(j_b p_b s_b) = 1$, $\pi(j_b p_b s_r) = \delta_1$, $\pi(j_b p_b s_w) = \delta_2$, $\pi(j_b p_w s_b) = \beta \cdot \delta_3$, $\pi(j_b p_w s_r) = \beta \cdot \delta_4$, $\pi(j_b p_w s_w) = \beta$, $\pi(j_r p_b s_b) = \alpha \cdot \delta_5$, $\pi(j_r p_b s_r) = \alpha$, $\pi(j_r p_b s_w) = \alpha \cdot \delta_6$, $\pi(j_r p_w s_b) = \pi(j_r p_w s_r) = \pi(j_r p_w s_w) = \alpha \cdot \beta$.*
Indeed, for instance, $\pi(j_r p_b s_b) = \pi(j_r) \cdot \pi(p_b) \cdot \pi(s_b|j_r p_b) = \alpha \cdot \delta_5$. Now, assume that the user considers the choice of the color of his pants as more important than the color of his shirt, then $\mathcal{C}$ is augmented with the additional constraint $\beta < \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6\}$. In this case, we can compare for instance $j_r p_b s_b \succ j_r p_w s_b$.

The preference specification is partial when the preference statements do not cover all the domains values of all the parent instantiations. A default principle, in case of missing information, may be to assume indifference, which amounts to assigning equal possibility degree to all corresponding options. From now on, we assume the complete specification of conditional preferences, i.e., in each possible context, the user provides a complete preordering of the values of the considered variable in terms of strict preference or indifference. As can be seen in the running example, our representation setting shares the same graphical structure as CP-nets [4]. But we are not adopting the worsening flips semantics of the latter, rather we use the chain rule and compare products of symbolic weights attached to solutions for defining the partial order between them.

2.4 Partial ordering induced by π-Pref nets

The purpose of preference modeling is to compare all possible solutions in Ω. Each possibility degree of a solution, computed from the product-based chain rule, expresses the satisfaction level of the solution. This leads to the following definition of the induced ordering.

Definition 3 Preference ordering*: Given a set of solutions Ω, a joint possibility distribution $\pi_{\Pi G}$ computed from a possibilistic preference network ΠG and a set $\mathcal{C}$ of constraints between the symbolic weights. Let ω_i and ω_j be two solutions of Ω. We have: (i) $\omega_i \succ \omega_j$ iff $\pi_{\Pi G}(\omega_i) > \pi_{\Pi G}(\omega_j)$; (ii) $\omega_i \sim \omega_j$ iff $\pi_{\Pi G}(\omega_i) = \pi_{\Pi G}(\omega_j)$; (iii) $\omega_i \pm \omega_j$ iff $\pi_{\Pi G}(\omega_i) \pm \pi_{\Pi G}(\omega_j)$, ($\pm$ denotes non comparability).*

Each solution $\omega = a_1 \dots a_N$ is associated with a vector $\vec{\omega} = (\alpha_1, \dots, \alpha_N)$, where $\alpha_i = \pi(a_i|u_i)$ and $u_i = \omega[Pa(A_i)]$. A natural ordering of such vectors is the Symmetric Pareto ordering $\succ_{SP}$, such that $\vec{\omega} \succ_{SP} \vec{\omega'}$ iff there exists a permutation σ of the components of $\vec{\omega'} = (\beta_1, \dots, \beta_N)$, yielding a vector $\vec{\omega'}_\sigma = (\beta'_1, \dots, \beta'_N)$, s.t. $\vec{\omega} \succ_{Pareto} \vec{\omega'}_\sigma$ (where $\vec{\omega} \succ_{Pareto} \vec{\omega'}_\sigma$ iff $\forall\, k,\ \alpha_k \geq \beta'_k$ and $\exists\, s$ s.t. $\alpha_s > \beta'_s$). The next proposition checks that the Symmetric Pareto ordering $\succ_{SP}$ on solutions is the same as the one induced by a product-based π-Pref net.

Proposition 1 *$\omega \succ_{SP} \omega'$ iff $\pi_{\Pi G}(\omega) > \pi_{\Pi G}(\omega')$.*

Proof (Informal) ($\Rightarrow$) This direction is obvious. ($\Leftarrow$) Assume that $\omega \succ_{SP} \omega'$ does not hold. If $\omega' \succeq_{SP} \omega$, then, clearly $\pi_{\Pi G}(\omega') \geq \pi_{\Pi G}(\omega)$. If $\omega \pm_{SP} \omega'$, then one possibility is that for each permutation, two pairs of components from each vector are ordered in opposite ways, another is that for each permutation, some components are incomparable. In each case, it is possible to find instantiations of the weights in such a way that their products leads to the domination of one vector over the other, and of the latter over the former. Hence the product ordering also yields incomparability.

3 Boolean π-Pref nets and their logical encodings

Boolean π-Pref nets are a particular case of interest. In this case, π-Pref nets can be equivalently expressed in terms of possibilistic logic, or penalty logic.

3.1 Agreement with the inclusion ordering in the Boolean case

If variables are *binary*, it is easy to define the violation of the preference statement associated to variable A_i by a solution. A solution ω violates the preference statement $u_i : a_{i1} > a_{i2}$ associated to variable A_i if and only if $\omega[Pa(A_i)] = u_i$ and $\omega[A_i] = a_{i2}$. A solution can violate only one preference statement per variable. Then an intuitive ranking of solutions is the inclusion ordering in the sense that if a solution ω violates all the preference statements violated by another solution ω' plus some other(s), then ω' is strictly preferred to ω. When no additional preference constraint is available, the ordering induced from the product-based π-Pref net boils down to this order.

Proposition 2 *Let ΠG be a possibilistic preference network with binary decision variables. Let ω, ω' be two solutions and $\pi_{\Pi G}$ be the joint possibility distribution induced from ΠG. Then ω falsifies all the preference statements falsified by ω' plus some other(s) if and only if $\pi_{\Pi G}(\omega) < \pi_{\Pi G}(\omega')$.*

Proof It is enough to notice that the Symmetric Pareto ordering then reduces to the inclusion ordering between subsets of violated preference statements.

Example 4 *Let V and W be two Boolean variables standing respectively for "vacations" and "weather" and these preference statements $w \succ \neg w$, $\neg w : v \sim \neg v$ and $w : v \succ \neg v$ (with w = 'good weather', v = 'having vacations'), giving birth to a π-Pref net ΠG: $\pi_{\Pi G}(w) = 1$, $\pi_{\Pi G}(\neg w) = \alpha$, $\pi_{\Pi G}(v|\neg w) = \pi_{\Pi G}(\neg v|\neg w) = 1$, $\pi_{\Pi G}(\neg v|w) = \beta$, $\pi_{\Pi G}(v|w) = 1$. We have $\pi_{\Pi G}(wv) = 1 > \pi_{\Pi G}(\neg wv) = \pi_{\Pi G}(\neg w \neg v) = \alpha$ and $\pi_{\Pi G}(wv) = 1 > \pi_{\Pi G}(w \neg v) = \beta$. Note that wv satisfies the two preference statements, while the other solutions only satisfy one. Moreover, $\neg w \neg v$ and $\neg wv$ satisfy the same preference statement. Thus, the ordering deduced from π-Pref net is indeed the same as the inclusion ordering.*

We should mention that although it is conjectured [9] that CP-nets are consistent with the inclusion order in the above sense, it was never formally proved.

3.2 Logical possibilistic encoding

Since the possibilistic setting offers different representation formats, π-Pref nets also have a logical counterpart offering another reading of the preferences, which may be of interest for reasoning purposes. Such a logical counterpart is a symbolic possibilistic base of the form $\Sigma = \{(f_1, c_1), \ldots, (f_m, c_m)\}$ which is a finite set of weighted formulas f_i where $c_i > 0$ is understood as a lower bound of a necessity degree $N(f_i)$ [8]. Its semantics is a possibility distribution $\pi_\Sigma(\omega) = min_{i=1,n} \pi_{\{(f_i,c_i)\}}(\omega) = 1$ if $\omega \vDash f_i$ and $1 - c_i$ if $\omega \vDash \neg f_i$. Each complete preorder on Ω can be represented by a possibility distribution. Moreover, any distribution can be associated with a possibilistic logic base, and also equivalently represented by a possibilistic network [3]. We now consider the possibilistic base associated to complete preference preorder at each node of the π-Pref net:

Definition 4 *The symbolic possibilistic base Σ_i associated to a Boolean variable A_i in a possibilistic network ΠG is defined as follows:*

- For each preference statement $u_i : a_{i1} \succ a_{i2}$ between the two possible values of a variable A_i, $(\neg u_i \vee a_{i1}, \beta) \in \Sigma_i$ where $\pi(a_{i2}|u_i) = 1 - \beta < 1$ in ΠG.
- There is no formula induced by preference statements $u_i : a_{i1} \sim a_{i2}$.

For Example 4, we get $\Sigma_W = \{(w, 1 - \alpha)\}$ and $\Sigma_V = \{(\neg w \vee v, 1 - \beta)\}$.

Proposition 3 *If π_i is the possibility distribution induced by Σ_i associated with node A_i, then $\pi_i(\omega[\{A_i\} \cup Pa(A_i)]) = \pi(a_i|u_i)$ where $a_i = \omega[A_i]$, $u_i = \omega[Pa(A_i)]$.*

Thus, $\pi_{\Pi G}(A_1, \ldots, A_N) = \times_{i=1,\ldots,N} \pi_i(\omega[\{A_i\} \cup Pa(A_i)])$.

The possibilistic base associated with a π-Pref net ΠG can be obtained by fusing the elementary bases Σ_i $(i = 1, ..., N)$ associated to its nodes. Since we are in the product-based setting, the combination of these possibilistic bases is defined iteratively as $Comb(\Sigma_1, \Sigma_2) = \Sigma_1 \cup \Sigma_2 \cup \{(p_i \vee q_j, \alpha_i + \beta_j - \alpha_i \times \beta_j) : i \in I,\ j \in J,\ p_i \vee q_j \neq \top\}$, where $\Sigma_1 = \{(p_i, \alpha_i) : i \in I\}$ and $\Sigma_2 = \{(q_j, \beta_j) : j \in J\}$. The base resulting from this product-based combination is a (possibly large) possibilistic base that encodes the same possibility distribution as $\pi_{\Pi G}$, see [8]. For Example 4 it reduces to $\Sigma_W \cup \Sigma_V$, as the third formula is a tautology.

3.3 Links with penalty logic

This subsection points out another logical counterpart of a π-Pref net ΠG (with distribution $\pi_{\Pi G}$), in terms of a penalty logic base PK [7], where weights are additive. More precisely, this logic associates to each formula the cost (in $[0, +\infty)$) to pay if this formula is violated. The penalty $k_{PK}(\omega)$ relative to a solution ω is the sum of the elementary penalties of the violated formulas. This contrasts with possibilistic logic, where weights are combined by an idempotent operation. The best solution has a cost equal to 0. This logic with a cost interpretation has a close relationship with product-based π-Pref nets. Indeed, the cost of a solution induced by a penalty logic base corresponds actually to the possibility degree computed from a π-Pref net. Namely, in each possibilistic base Σ_i associated to a node A_i we can at most violate one formula. Thus, for each possibilistic base $\Sigma_i = \{(f_{i1}, \alpha_{i1}), \ldots, (f_{ik}, \alpha_{ik})\}$ there exists a penalty logic base $PK_i = \{(f_{i1}, -\ln(\alpha_{i1})), \ldots, (f_{ik}, -\ln(\alpha_{ik}))\}$ such that the ordering induced by π_i is the same as the order induced by the cost function of the penalty logic. This mirrors the fact that $\pi_{\Pi G}(\omega) = \alpha_1 \cdot \ldots \cdot \alpha_N \Leftrightarrow\ k_{PK}(\omega) = -(ln(\alpha_1) + \ldots + ln(\alpha_N))$. Contrarily to possibilistic bases, the combination between penalty bases is the union of all PK_i $(i = 1, ..., N)$. This yields the same ordering as π-Pref nets. But there is no proof system for penalty logic yet.

4 Optimization and dominance queries

In π-Pref nets, conditional preferences correspond to nodes associated with conditional possibility tables. We restrict ourselves to π-Pref nets that are Directed Acyclic Graphs (DAG). On this basis and using the chain rule, one can compute the symbolic possibilities of completely instantiated alternatives, which can then be compared. Two types of queries are usually considered: Optimization queries (for finding the optimal solution), and dominance queries (for comparing solutions). We now study the two types of queries are presented.

4.1 Optimization

For acyclic CP-nets, the optimization query is linear in the size of the network (using a forward sweep algorithm), and there is always a unique optimal solution [4]. In our case, this query may return several solutions since, contrarily to CP-nets, we allow the user to express indifference. Clearly, the best solutions are those having a joint possibility degree equal to 1. Indeed, such a solution exists since the joint possibility distribution associated to the possibilistic network is normalized, thanks to the normalization of each conditional possibility table (i.e. for each variable A_i, each instantiation u_i of $Pa(A_i)$: $\max(\pi(a_i \mid u_i), \pi(\neg a_i \mid u_i)) = 1$ where $\{\neg a_i\} = D_{A_i}/\{a_i\}$ with $a_i \in D_{A_i}$). Thus, we can always find an optimal solution, starting from the root nodes where we choose each time the most or one of the most preferred value(s) (i.e. with possibility equal to 1). Then, depending on the parents instantiation, each time we again choose an alternative with a conditional possibility equal to 1. At the end of the procedure, we get one or several completely instantiated solutions having a possibility equal to 1. Consequently, partial preference orders with incomparable maximal elements can not be represented by a π-Pref net.

Example 5 *Let us reconsider Example 2 and its joint possibility degree in Example 3. Then, $j_b p_b s_b$ is the preferred solution since its joint possibility is equal to* 1*, and this is the only one.*

The complexity of optimization queries in possibilistic networks is the same as the CP-nets forward sweep procedure if the network omits indifference. In a more general case where indifference is allowed, we can use the same principle as when searching for the best explanations in Bayesian networks [6]. In fact the Most Probable Explanations (MPE) can be obtained by adapting the propagation algorithm in junction trees [12] by replacing summation by maximum. This algorithm has the same complexity as probability propagation (i.e. NP-hard) except in the particular case when the DAG is a polytree since the MPE task can be accomplished efficiently using Pearl's polytree algorithm [14]. The adaptation of this algorithm for the possibilistic framework can be easily performed on the product-based Junction tree algorithm [2] with the same complexity as the standard MPE. A possible variant of the optimization problem is to compute the M most possible configurations using a variant of the MPE [13]. This query is not proposed in CP-nets and can be interesting in π-Pref nets even if the answer is not always obvious to obtain in presence of incomparable solutions.

4.2 Dominance

The comparison between the symbolic possibility degrees can be found using Algorithm 1.1 that takes as input the set of constraints $\mathcal{C}$ between the symbolic weights and two vectors. Let us consider two solutions ω_i and ω_j with simplified respective vectors $\overrightarrow{\omega_i^*} = (\alpha_1, \ldots, \alpha_k)$ and $\overrightarrow{\omega_j^*} = (\beta_1, \ldots, \beta_m)$ where the components equal to 1 have been deleted, with $k \leq m \leq N$. Then, the algorithm proceeds by first deleting all pairs of equal components between the vectors so to get totally different components. Second, if there exists a permutation where each

component α_i is higher than β_s such that $s \in [1, ..., k]$ then $\omega_i \succ \omega_j$, otherwise they remain non comparable. Thus the algorithm is based on the sequential application of:
(1) The function *equality* that deletes the common values between $\vec{\omega_i}$ and $\vec{\omega_j}$.
(2) The function *sort* that returns *true* if given $\alpha_c \in \vec{\omega_i}$, there exists a constraint $\alpha_c > \delta$ in $\mathcal{C}$ such that $\delta \in \vec{\omega_j}$. Each component of $\vec{\omega_j}$ can be used only one time in the comparison process.

Algorithm 1.1. Comparison between two joint possibility degrees

Data: $\vec{\omega_i}$, $\vec{\omega_j}$, $\mathcal{C}$
Result: R
begin
  equality($\vec{\omega_i}$, $\vec{\omega_j}$, $\mathcal{C}$);
  if *(empty*($\vec{\omega_i}$) *and empty*($\vec{\omega_i}$)) **then** $R \leftarrow \omega_i = \omega_j$; **else** $s \leftarrow true$;
  $s \leftarrow sort(\vec{\omega_i}, \vec{\omega_j}, \mathcal{C})$;
  if $s = true$ **then** $R \leftarrow \omega_i \succ \omega_j$;
  else $R \leftarrow \omega_i \pm \omega_j$;
  return R
end

Example 6 *Let us consider the* π*-Pref net* ΠG *of Example 2. Using Algorithm 1.1, the ordering between the solutions is defined in Figure 2 such that a link from* ω_i *to* ω_j *means that* ω_i *is preferred to* ω_j*. For instance, consider* $\overrightarrow{j_b p_w s_r} = (\beta,\ \delta_4)$ *and* $\overrightarrow{j_r p_w s_r} = (\alpha,\ \beta)$*. First, we should delete common values, namely the symbolic weight* β*. Then, we should check if* $\mathcal{C}$ *entails* $\alpha < \delta_4$ *or the inverse. Here,* α *and* δ_4 *are not comparable. Thus, we have* $j_b p_w s_r \pm j_r p_w s_r$*.*

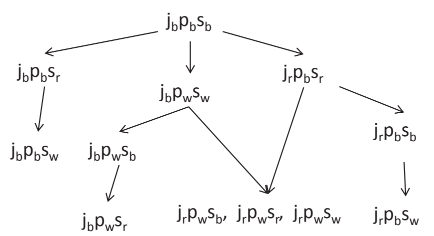


Fig. 2. Possibilistic order relative to Example 2

The complexity of dominance in CP-nets depends on the network structure. For singly connected binary-valued CP-nets it has been proved that the problem is NP-complete (using a reduction to 3SAT). In the general case [10] shows that it is a PSPACE-complete. Clearly, for π-Pref nets, the complexity is due to the comparison step in Algorithm 1.1 (since the computation of the possibility degrees is a simple matter using the chain rule) and in particular to the *sort* function where the matching between the two vectors needs the definition of different possible arrangements i.e. the algorithm is of time complexity O(n!).

5 Conclusion

This paper has established the main properties of possibilistic conditional preference networks. This modeling is appropriate to represent conditional preferences without having the CP-nets limitations, namely the enforced priority in favor of parent nodes. Moreover, we have shown that π-Pref nets produce a symmetric Pareto ordering of solutions, and in the Boolean case are endowed with logical counterparts allowing an equivalent modeling suitable for inference.

This work calls for several developments. In fact, we might think of partially specified preferences as well as the handling of impossible situations. Also, it would be interesting to conduct a deep comparison with other preference models such as GAI networks [11] and UCP-net [5] since they both use additive utilities.

References

1. N. Ben Amor, D. Dubois, H. Gouider, and H. Prade. Possibilistic networks: A new setting for modeling preferences. In *8th Int. Conf. SUM*, pages 1–7. Springer, 2014.
2. N. Ben Amor, S. Benferhat, and K. Mellouli. Anytime possibilistic propagation algorithm. In *Proc. 1st Int. Conf. Comput. in an Imperfect World*, pages 263–279, 2002.
3. S. Benferhat, D. Dubois, L. Garcia, and H. Prade. On the transformation between possibilistic logic bases and possibilistic causal networks. *Int. J. Approx. Reasoning*, 29(2):135–173, 2002.
4. C. Boutilier, R. I. Brafman, C.l Domshlak, H. H. Hoos, and D. Poole. CP-nets: A tool for representing and reasoning with conditional ceteris paribus preference statements. *J. Artif. Intell. Res*, 21:135–191, 2004.
5. R. I. Brafman and C. Domshlak. Introducing variable importance tradeoffs into CP-nets. *CoRR*, 2013.
6. A. P. Dawid. Applications of a general propagation algorithm for probabilistic expert systems. *Statistics and Computing*, 2(1):25–36, 1992.
7. F. Dupin de Saint-Cyr, J. Lang, and T. Schiex. Penalty logic and its link with Dempster-Shafer theory. In *Proc. 10th conf. UAI*, pages 204–211. Morgan Kaufmann, 1994.
8. D. Dubois and H. Prade. Possibilistic logic: a retrospective and prospective view. *Fuzzy Sets and Systems*, 144(1):3–23, 2004.
9. D. Dubois, H. Prade, and F. Touazi. Conditional Preference-nets, possibilistic logic, and the transitivity of priorities. In *Proc. 33rd SGAI Int.l Conf., Cambridge*, pages 175–184. Springer, 2013.
10. J. Goldsmith, J. Lang, M. Truszczynski, and N. Wilson. The computational complexity of dominance and consistency in CP-nets. *J. Artif. Intell. Res.*, pages 403–432, 2008.
11. C. Gonzales and P. Perny. GAI networks for utility elicitation. In *Proc. 9th int. conf. Principles of Knowledge Representation and Reasoning*, pages 224–234, 2004.
12. F. V. Jensen, S. L. Lauritzen, and K. G. Olesen. Bayesian updating in causal probabilistic networks by local computations. *Computational Statistics Quarterly*, 4:269–282, 1990.
13. D. Nilsson. An efficient algorithm for finding the m most probable configurations in probabilistic expert systems. *Statistics and Computing*, 8(2):159–173, 1998.
14. J. Pearl. *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. Morgan Kaufmman, 1988.